A Cauchy–Euler ODE is in the form

$$A_n(x) \cdot x^n \cdot y^{(n)}(x) + \cdots + A_2(x) \cdot x^2 \cdot y''(x) + A_1(x) \cdot x \cdot y'(x) + A_0(x) \cdot y = G(x).$$

and may be converted to a linear ODE with constant coefficients by making the substitution $x = e^t$.

1. If $x = e^t$, find $\frac{dy}{dx}$ and $\frac{d^2 y}{dx^2}$

Solve each equation.

2. $x^2 y'' - 2xy' + 2y = x^3, \quad x > 0$

3. $x^2 y'' + 3xy' - 8y = \ln^3 x - \ln x, \quad x > 0$
Differential Operator

\[ D = \frac{d}{dx} \] is a linear differential operator.

\[ D(x^2) = 2x, \quad D(e^{3x}) = 3e^{3x}, \quad \text{and} \quad D(\sin x) = \cos x \]

A differential operator can annihilate functions as shown below

\[ D^n \text{ annihilates each of } 1, x, x^2, \ldots, x^{n-1} \]

\[ (D - \alpha) \text{ annihilates each of } e^{\alpha x}, xe^{\alpha x}, e^{\alpha x^2}, \ldots, e^{\alpha x^{n-1}} \]

\[ [D^2 - 2\alpha D + (\alpha^2 + \beta^2)]^n \text{ annihilates each of } \\
e^{\alpha x} \cos \beta x, xe^{\alpha x} \cos \beta x, e^{\alpha x^2} \cos \beta x, \ldots, e^{\alpha x^{n-1}} \cos \beta x \\
e^{\alpha x} \sin \beta x, xe^{\alpha x} \sin \beta x, e^{\alpha x^2} \sin \beta x, \ldots, e^{\alpha x^{n-1}} \sin \beta x \]

4. Verify that \( 2D - 1 \) annihilates \( y = e^{x/2} \)

5. Find the differential operator that annihilates \( x^3(1 - 5x) \)

6. Find the linearly independent functions that are annihilated by the given differential operators.
   a) \( D^2 + 4D \)
   b) \( D^2 - 6D + 10 \)
   c) \( D^2(D - 5)(D - 7) \)

7. Solve the system by using Differential operators and systematic elimination.

\[ \frac{dx}{dt} + \frac{dy}{dt} = e^t \]

\[ - \frac{d^2x}{dt^2} + \frac{dx}{dt} + x + y = 0 \]