Exact ODE:
A 1st order ODE in the form $M \, dx + N \, dy = 0$ is Exact if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

To solve an Exact ODE,
1. let $F(x, y) = \int M \, dx$
2. Integrate $M$ with respect to $x$ and use $g(y)$ as the constant.
3. Differentiate the result with respect to $y$; the constant $g(y)$ becomes $g'(y)$.
4. Set this result equal to $N$ to determine $g'(y)$.
5. $\int g'(y) \, dy$ to find the value of $g(y)$. This value does include a constant.
6. Substitute this value into the previous expression containing $g(y)$ and you have the solution, $F(x, y)$.
7. The general solution is formed by setting the constant equal to the remaining expression.

Use this method to determine if the following are Exact ODE’s. If they are, solve the equation.

1. $\frac{y}{x^2} \frac{dy}{dx} + e^{2x+y} = 0$
2. $\left( x^2 + \frac{2y}{x} \right) \, dx = (3 - \ln(x^2)) \, dy$
3. $(6x + 1)y^2 \frac{dy}{dx} + 3x^2 + 2y^3 = 0$

Homogeneous 1st–order ODE:
$\frac{dy}{dx} = f(x, y)$ is a 1st–order homogeneous ODE if $f(tx, ty) = t^n f(x, y)$ for some constant $n$.

Homogeneous 1st–order ODE’s may be converted to separable ODE’s by making either of the following substitutions: $u = \frac{y}{x}$ or $v = \frac{x}{y}$

Use this method to determine if the following are Homogeneous ODE’s. If they are, solve the equation.

4. $\frac{dy}{dx} = \frac{y^2 + 2xy}{x^2}$
5. \[ \frac{dy}{dx} = \frac{x}{y} + \frac{y}{x} + 1 \]

Bernoulli 1st-order ODE:
\[ \frac{dy}{dx} + P(x)y = Q(x)y^n \] is the form for a 1st-order Bernoulli ODE if \( n \) is not equal to either 0 or 1.

Bernoulli 1st–order ODE’s may be converted to Linear ODE’s by making the following substitution: \( u = y^{1-n} \)

Use this method to determine if either of the following are Bernoulli ODE’s. If it is, solve the equation.

6. \[ \frac{dy}{dx} - 5y = y^2 \]
7. \[ \frac{dy}{dx} = \frac{y^2 + 2xy}{x^2} \]

Euler’s Method.
Euler’s Method may be used to approximate the value of \( y_{n+1} \), based upon a previous value of \( y_n \), and the slope of the function. The approximation is given by \( y_{n+1} = y_n + hf(x_n, y_n) \), where \( h \) is called the step size.

8. The differential equation used to approximate the concentration of a drug into a patient’s bloodstream is \( \frac{dC}{dt} = Ce^{-t} \). If the initial drug concentration is 0.025 \( \mu \)g/ml. Use Euler’s method to predict the concentration of drug in the patient’s bloodstream after 1 hour, \( C(1) \). Let \( \Delta t = 0.5 \).